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# SVD Based Optimal Massive MIMO Transmission and Capacity

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***Abstract*— Spatial multiplexing of channel is the core of diversity in Multiple Input Multiple Output (MIMO) and massive MIMO. The array of antennas on a base station is the key to success for upcoming 5G multimedia communication network. Massive MIMO due to its substantial number of antennas produces high data rate. when an UEs transmit a signal towards base stations, large number of parallel streams produce high capacity. Channel interference and small-scale fading is minimized. In this paper we will analyze the massive MIMO model for single antenna MS transmit signal towards BS having hundreds of antennas. We will examine the behavior of this uplink scenario using matrix theory. We will develop SVD based massive MIMO system and investigate how SVD based system provides interactive feature of spatial multiplexing and decoupling of channels. Our hypothesis will prove minimization of interference and fading. We will examine the total capacity of parallel stream. This performance analysis will prove an optimal SVD based massive MIMO model**

***Index Terms*—Channel Estimation, MIMO, Spatial Multiplexing, SVD, ZF Receiver.**

## I. INTRODUCTION

Massive MIMO multimedia communication network design is described as a base station (BS) with hundreds of antennas instantaneously serve a set of single-antenna MSs and the multiplexing gain can be shared by all MSs. The cheap antennas are supposed to be used in massive MIMO instead of costly antenna system. Due to its diversity nature, the massive MIMO is less sensitive to the propagation environment than in the conventional MIMO case [2]. Massive MIMO has become an essential part of Wireless communications standards, such as 802.11 (WiFi), 802.16 (WiMAX), LTE. The illustration of massive MIMO network is given in Fig1. [1]

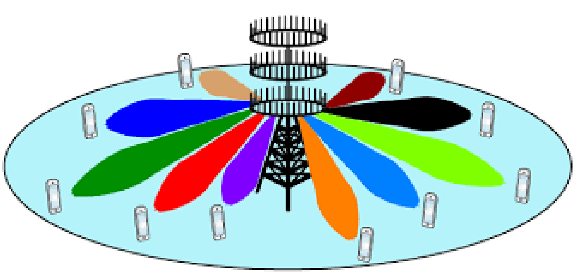

Figure 1: illustration of massive MIMO wireless network [4].

It is observed that, as the number of antennas increase the effects of uncorrelated noise and small-scale fading eliminate [3]. The concept of deep fading is finished in massive MIMO due to its parallel stream and LOS is maintained. The number of users is said to be independent of the size of cell. The antenna structure in massive MIMO supports the low energy consumption and negligible channel interference due to decoupling of channels. the received signal is approximately same as transmitted. [2]

The linear receivers like Maximum Ratio Combining (MRC), Zero Forcing (ZF), Minimum Mean Square Error (MMSE) produce excellent results unlike conventional MIMO performance which is only proportional to high SNR. signal processing approaches, such as matched- filter (MF) precoding/detection, is used in massive MIMO systems [6][8]. The assumption for MF based massive MIMO network is shown. [1][3] that under realistic propagation 17 Mb/s data rate could be achieved for each of 40 users in a 20 MHz channel with an average throughput of 730 Mb/s per cell and an overall spectral efficiency of 26.5 bps/Hz. Since the number of antennas at the BS assumed to be significantly larger [2] [11] [22].

In massive MIMO number of antennas on base station are larger than mobile station (MS) antennas that is why the channel matrix is not invertible like square matrix so instead of simple decomposition methods some complex methods are used like, Jacobi and Gauss-Seidel methods etc. [3] [13] In our proposed work, we will analyze the SVD based massive MIMO. The singular Value Decomposition (SVD) method produce simple decomposition in to parallel stream in case of linear receivers. The spatial multiplexing performs a tremendous role in massive MIMO and decoupling of channel decreases the channel interference. This paper is distributed in following section. in Section II we will discuss system model of our prescribed scenario of massive MIMO wireless communication network. section III will explain singular value Decomposition (SVD)method. We will analyze SVD based Massive MIMO system and compute total capacity in Section IV. In Section V, Conclusions will be drawn. [6]

Notations: In this paper, upper/lowercase boldface letters denote matrices/ vectors. $\mathrm{X_i}, \mathrm{h_{ji}}$, and y 1,2…..m denote the input signal, channel matrix and output signals respectively the {h} h and{t} shows the Hermitian and transpose . $\mathrm{Ex}, \mathrm{Ex}^2, \mathrm{Rx}^2$ describe the expected value , variance and covariance. I represent identity matrix, M describe the large number of antennas placed on BS and K shows the single MS antenna. The acronym i.i.d denote independent and identically distributed, respectively.

## II. SYSTEM MODEL

we will discuss our proposed scenario which is an uplink massive MIMO network model given in Fig 2, where the BS contain M antennas Where $\mathrm{j_1}, \mathrm{j_2} \ldots . \mathrm{j_m}$ m are serving on BS and K are the mobile stations antennas describe as $\mathrm{i_1}, \mathrm{i_2} \ldots \ldots . \mathrm{i_k}$ . Each MS contain single antenna. The scenario is considered in our study which can be designed as M >>K Here N ≤ K, is likely pilot design approach. We also consider Rayleigh communication fading model $f_{A(a)} = 2\mathrm{ae}{-}^{2}$ [6]


Figure 2: massive MIMO wireless network model

Our proposed scenario is a single antenna I from K set of mobile station antennas is transmitting a symbol $\mathrm{x_i}$ which is received by M number of antennas placed on BS which are $\mathrm{M} = \mathrm{j_1}, \mathrm{j_2} \ldots \mathrm{j_m}$ the output signals will be received on BS which would be $\mathrm{y_1}, \mathrm{y_2}, \mathrm{y_3} \ldots \mathrm{y_m}$ .The basic equation can be expressed as

$$\mathrm{Y} = \mathrm{HX} + \mathrm{N} \quad (1)$$

"Y" term as received signal and "X" is transmitted signal where "H" denotes the channel Coefficient and "N" is additive white Gaussian noise (AWGN). Now putting the values in (1). The output signal Y can be expressed as

$$\mathrm{Y} = \{\mathrm{h_{j_{1_i}}}, \mathrm{h_{j_{2_i}}} \ldots . . \mathrm{h_{j_{m_i}}}\}^{\mathrm{T}}\{\mathrm{x_i}\} + \{\mathrm{n_{j_1}}, \mathrm{n_{j_2}} \ldots . \mathrm{n_{j_m}}\}^{\mathrm{T}} \quad (2)$$

The signal is transmitted in a time instant "t "as we have considered time domain, so the output signal can be expressed as,

$$\mathrm{Y} = \textstyle\sum_{\mathrm{j=1}}^{\mathrm{m}} \quad \mathrm{h_{ji}x_i} + \mathrm{n_j} \qquad (3)$$

This is clearly shown that a signal is transmitting from a mobile station and a large number of antennas are receiving this signal. [8] [16] Zero forcing (ZF) receivers are supposed to be used on base station and transmitting signal is precoding before transmitting [7] [16]. we are going to decompose the channel matrix using singular value decomposition method for our given scenario. We have discussed that a single antenna is transmitting a signal and large number of antennas on base station are receiving that signal. We will analyze the performance of SVD based massive MIMO. We will prove how spatial multiplexing produce independent stream with help of matrix theory. We will compute the expression for total capacity of our prescribed scenario of massive MIMO multimedia network [5][20][14]

## III. SVD

Singular value decomposition is very useful method for characterization of channels and it is popular in analyzing the behavior of MIMO and massive MIMO wireless communication. It produces the decomposition of channel matrix. Let us consider channel matrix as given above r × t which is stated as r > t the SVD is described as given bellow [1] [14] [16],

$$\mathrm{H} = \mathrm{U\Sigma V^H} \qquad (4)$$

Here U is r×t matrix having t column and Σ is diagonal matrix containing singular value. It is t×t matrix and V is unitary matrix also t × t matrix as well. SVD is implied only if the properties of SVD are followed which are given bellow [4] [16].

$$u_1^h u_2 = \begin{cases} 1 & if \ i = j \\ 0 & if \ i \neq j \end{cases} \qquad (5)$$

The column should be orthonormal. It concluded that

$$\mathrm{U^H U} = \mathrm{I_{t \times t}} \qquad (6)$$

Second property regarding V matrix which is stated as:

$$V^H V = VV^H = I_{t\times t} \qquad (7)$$

This matrix v is known as unitary matrix Third property for singular value matrix which is stated as

$$\sigma_1, \sigma_2, \dots\dots..\sigma_n > 0 \qquad (8)$$

$$\sigma_1 > \sigma_2 > \cdots \dots.>.\sigma_n \qquad (9)$$

Singular values are non-negative and are arranged in decreasing order number of nonzero singular value denote the rank of matrix in next Section, we will analyze the performance of our prescribed scenario using singular value decomposition methods and drive the expression for SVD and find out output signal Y. We will compute and analyze the expression for total capacity for parallel stream of channel [3-5].

## IV. PERFORMANCE ANALYSIS

Let us start to find out the expression. We will substitute the value in SVD according to our scenario and the equation can write as [12-13],

$$U\Sigma V^H = \{1\}^H \qquad (10)$$

U vector is normalized as unit norm and singular value matrix is t × t matrix and V matrix is also in same formation now we can see that all the properties of SVD is followed for the substitution of values in $U^H U - I$ and Σ matrix is non-zero matrix. It shows that the expression of our prescribed scenario is valid for SVD decomposition although if the expression does not follow all the properties then using row matrix [6][16]19] operation we must convert the expression so that it follows the properties of SVD. The massive MIMO channel matrix has derived as $H = U\Sigma V^h$ using singular value decomposition method we know the system modal equation as given Eq[1].which can be write as

$$\overline{Y} = U^H \overline{x} + \overline{n} \qquad (11)$$

At the receiver, we multiply $U^H$ which can be written as

$$U^H \overline{Y} = U^H (U^H \overline{x} + \overline{n}) \qquad (12)$$

$U^H \overline{Y}$ can be demoted as Y the above equation can be rewritten as

$$\tilde{Y} = \Sigma V^h x + U^H \bar{n} \qquad (13)$$

The value of $U^H \bar{n}$ can be replaced some modified noise vector which is term as n this effective noise vector Now some manipulation on the transmitter as called precoding It must be done before transmission $\bar{x} = Vx$ here is transmit vector it effects on our system as given in equation [21-22]

$$\widetilde{Y} = \Sigma\tilde{x} + \tilde{n} \qquad (14)$$

However, we know Σ is diagonal matrix Here if we elaborate our equation and observe that it looks as given bellow in Eq[15]. We are observing in this scenario there is no interference between symbol. This transform domain is also called decoupling of massive MIMO channel and known as parallelization of massive MIMO system [4] the equation can be expanded as

$$\widetilde{y_1} = \sigma\widetilde{x_\iota} + \widetilde{n_1}$$
$$\widetilde{y_2} = \sigma\widetilde{x_\iota} + \widetilde{n_2}$$
$$\widetilde{y_m} = \sigma\widetilde{x_\iota} + \widetilde{n_m} \qquad (15)$$

This is the collection of parallel channels this is possible due to singular value decomposition system in which A symbol is added with the signal on transmitter end to increase SNR. This transformation of signal is called beamforming and precoding schemes are applied on receivers for getting original signal. A symbol is transmitted through parallel channel which termed as spatial multiplexing, stated as in same time same frequency [3] [12] [14]. It can be written as,

$$\tilde{n} = U^H n$$
$$E\{\widetilde{n\, n^h}\} = E\{U\, \widetilde{n\, n^h} U^h\}$$
$$= \sigma_n^2 I \qquad (16)$$

It is shown that noise power is uncorrelated in different antennas as shown (17), which mean power of noise before beam forming and after beamforming now we can compute SNR of these parallel channel. Let us see the SNR of one channel as jth,

$$SNR = \{\sigma_j^2 p_j)/(\sigma_n^2\} \qquad (17)$$

Now we can investigate the SNR of parallel channel it can be understand with the diagram given bellow,

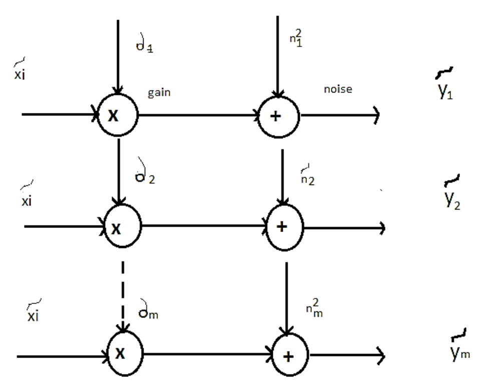


Figure 3: stream of parallel decoupled channel

The parallel channel which is shown the schematic diagram the independent stream of symbol transmitted through parallel channel which is spatial multiplexing [6, 16]. Now t we will see the Shannon capacity for maximum rate which is given by [5, 11].

$$C = Blog_2(1 + SNR) \qquad (18)$$

Putting the value of SNR of jth channel

$$C = Blog_2\left(1 + \sigma_j^2 p_j / \sigma_n^2\right) \qquad (19)$$

$$C = B \sum_{j=1}^{m} log_2\left(1 + \sigma_j^2 p_j / \sigma_n^2\right) \qquad (20)$$

The sum of individual capacity of each parallel stream shows that noise factor and interference decreases as the number of parallel stream increases this is leads to increase the net capacity of wireless communication network. This is one of the motivation behind the recognition of massive MIMO as supposed to be core technology of upcoming 5G multimedia communication network.

## V. CONCLUSION

SVD based massive MIMO offers the optimal transmission and maximum channel capacity near about 10 time of conventional capacity. Our hypothesis also proved that SVD based system provides interactive feature of spatial multiplexing and decoupling of channel which clearly shows the negligible interference and fading. in short, the SVD produces simple and interactive decomposition and optimal performance for massive MIMO. The massive MIMO would be the key for the upcoming 5G wireless communication network.

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