

Retinal Blood Vessels Segmentation Using Local Ternary Pattern

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Abstract- Retinal vessels segmentation is an important task in medical image analysis. This paper presents a robust vessel segmentation algorithm. The proposed method initially, isolates the Green channel from the input image. On Green channel, Local Ternary Patterns (LTP) is applied to obtain the robust vessel features. To remove noise from the small vessel features, multiple morphological filters, such as spur, majority, opening, and fill operations are used and finally we get the prominent vessels pixel and segmented image. The proposed LTP based vessel segmentation algorithm outperforms recently published methods in terms of Accuracy in some cases on DRIVE and STARE datasets. The proposed method shows a noteworthy and precise addition for imminent computer-aided diagnosis applications.

Index Terms-- Retinal Images, Segmentation, Vessels.

I. INTRODUCTION

An automatic and robust vessels segmentation in retinal images is a vital task by means of a Computer Aided Diagnostic (CAD) system to screen the Diabetic Retinopathy (DR). The DR is the obstacle of diabetes and major source of blindness [1]. Automatic, accurate, and robust vessel segmentation can assist in numerous purposes in such CAD systems. In later stages, the vessels may get distorted due to high blood pressure. To detect these distortions, constant monitoring and a precise vessel segmentation is desired. Moreover, diagnosis of DR is highly desirable in earlier stages to stop the progression and to secure the vision loss. Vessel segmentation is a challenging problem even for clear images due to highly complex nature of vasculature network. However, it gets more complicated as the images reveal pathological symptoms, such as bright lesions. Therefore, an accurate segmentation is essential, which does not require experts. Due to current advances in image processing algorithms developments, an improved and reliable segmentation is obtained [2,3]. In practical life, a robust and automated algorithm with minimum execution time along with requirement of least configuring parameters setting is the need of the day [4]. Therefore, encouraged from these facts, our key contributions are listed below.

- We present a robust and accurate unsupervised vessel segmentation algorithm, which is primarily based on intelligent combination of the Local Ternary Patterns (LTP) along with the morphological and Frangi filters. The LTP is suitable to extract features of thin vessels and small branches in the retinal image. To the best of our information, none of the earlier published works use the LTP to achieve segmentation.

- We also investigate the computational complexity of our proposed algorithm on various image resolutions. We are optimistic that our developed LTP based vessel segmentation algorithm will be handy for physicians to timely diagnose various illness associated with the veins. Moreover, we believe that our proposed algorithm will be used in complex structures medical images.

The paper is organized as follows. Section II presents recent developments in vessel segmentation. Section III describes proposed algorithms. Simulation results are discussed in Section IV and conclusions are listed in Section V.

II. RELATED WORK

Below, we briefly discuss recent vessel segmentation developments. In [5], researchers proposed a deep learning model in which the task is divided into three stages, which are vessel fusion, segmentation of thin vessels, and thick vessels. This work minimizes the negative effects, which are produced due to imbalance ratio. The results are refined in the final stage. Published work in [6] discussed a novel algorithm for segmentation that overcomes the difficulty while dealing with the challenging situations by applying enhancement method and then processing that image in the U-Net. In [7], published work presents a segmentation technique on the basis of multi-classifier fusion and multiple features. Moreover, in this work, contrast enhanced intensity, line strength, fused gray-voting, and B-COSFIRE filter response are four key features that are extracted. For decision of classifier, the results of AdaBoost algorithm classifier and decision tree are fused. In [8], published work presents a model, which addresses issues of extreme variations in the morphology of the vessels. For weighted attention method and dealing with small thin vessels, this model skipped connection scheme. In [9], published segmentation

methods uses a combination of Generative Adversarial Network (GAN) and U-Net model for segmentation. Their work reports higher accuracy than few of the methods compared therein. Sheng et al. [10] proposed a vessel segmentation method, which primarily focuses to segment narrow and low contrast vessels. The concept of super pixel as an elementary unit is also used to robustly segment retinal vessels. Regularization of that scheme is done by the combination of shape, texture and color. The refining of results is done for the detection of the local and global structures on vessel images by using minimum spanning super pixel tree. In [11], researchers presented a novel method to extract blood vessels from fundus images. This approach removes over bright lesions and provides a new feature for ellipse recognizing vessel. In [12], proposed method combines multiple results of segmentation to get an accurate result. The work reported in [13] discusses a technique that achieves vessel segmentation in various step. Firstly, the image is pre-processed and enhanced. Finally, multi-scale line detector is used for detection of vessels. In [14], the published work addresses the drawbacks that occurred during RVS by using differential image methods and the use of Primal-Dual Asynchronous Particle Swarm Optimization (PDAPSO). Khomri et al. [15] developed a technique, which they refer EMOABC calculates thresholding criteria to reduce noise by using energy curve function to separate vessels from the background. Parameters of the EMOABC algorithm are adjusted by using stopping criterion method. This scheme is computationally efficient than the compared methods. In [16], a robust technique is reported by utilizing fuzzy sets and C-means clustering. The contrast Enhancement of retinal vessels is done by using CLAHE and noise is reduced by mathematical morphologies. This work primarily uses Gabor and Frangi filters to enhance vessels network. For segmentation fuzzy C-means is used and refinement is done using integrated level set approach. Yue et al. [17], proposed a multi-scale line detector technique that determines line responses of vessels in windows of multi-scale and then takes maximum to get response values so that pale vessel pixels responses are enhanced that are in close to dark background pixels or strong pixels. In [18], a robust vessel segmentation technique is proposed achieves vessels extraction through histogram-based processing methods and Otsu thresholding. To remove optic disc and macula, morphological top-hat filters are used. For rejecting misclassified vessel pixels, a Vessel Location Map (VLM) is extracted in final stages. The VLM and Frangi filters operate in final stages to obtain the segmented image. In [19], proposed method reports vessels segmentation mainly through hysteresis thresholding, morphology, and top-hat transform. Dash et al. [20] developed a technique in which vessels are segmented by CLAHE, morphological cleaning operations, and gamma correction methods.

Few of the above described are nice efforts to segment vessels. However, we believe that our proposed LTP based segmentation method is one of the new additions in the field. In the next section, we discuss the developed vessel segmentation algorithm.

III. PROPOSED ALGORITHM

This section concisely presents our developed LTP based segmentation method. For clear understanding, the proposed method is divided into following interconnected steps. FIGURE 2 shows the flow of our developed algorithm.

A. IMAGE ACQUISITION

This step is fundamentally the pre-processing on the colored RGB image. We isolate the input colored Red, Green, Blue retinal image into the *RGB* channels as described in (1).

$$I = (R, G, B) \quad (1)$$

Where(*I*) is the isolated image. We process only Green channel in our work.

B. THE LOCAL TERNARY PATTERN (LTP)

The LTP is the advanced version of Local Binary Patterns (LBP). In our work, we choose to apply the LTP, because LTPs are less sensitive to noise [26]. Moreover, the gray level transformation LTP is not invariant. Unlike LBP, the LTP does not thresholds pixel values into 0 and 1. Rather it thresholds pixels into three valued code. Consider threshold constant (*t*), center pixel (*c*), and the neighboring pixel (*p*), so that the threshold result is:

$$LTP_{[p,c,t]} = \begin{cases} 1, & \text{if } p > c + t \\ 0, & \text{if } p > c - t \text{ and } p < c + t \\ -1, & \text{if } p < c - t \end{cases} \quad (2)$$

We select a 3×3 image patch and a threshold (*t*). When the threshold is between *c - t* and *c + t*, the pixel value assigned is 0. The pixel value assigned is 1 when *p > c + t* and is assigned -1 when *p < c - t*. When ternary codes are determined, they are split up into upper and lower patterns. The upper patterns are obtained by replacing the negative values by 0. Therefore, function is modified and at the output we have both lower and upper patterns. The LTP's computation is shown with the help of an example in FIGURE 1 with *t = 5*. Once the LTPs are computed, then multiple morphological filters are used.

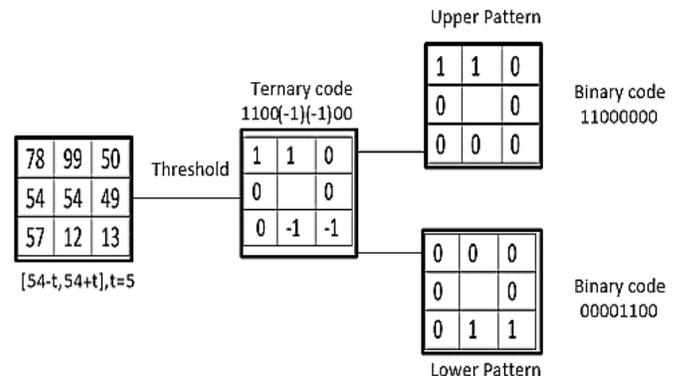


FIGURE 1: THE LTP COMPUTATION PROCEDURE

C. MULTIPLE MORPHOLOGICAL OPERATIONS

After application of LTP multiple morphological operations such as spur, majority, opening and fill operations are used to eradicate geometrical objects from the vessels image. We will discuss them one by one.

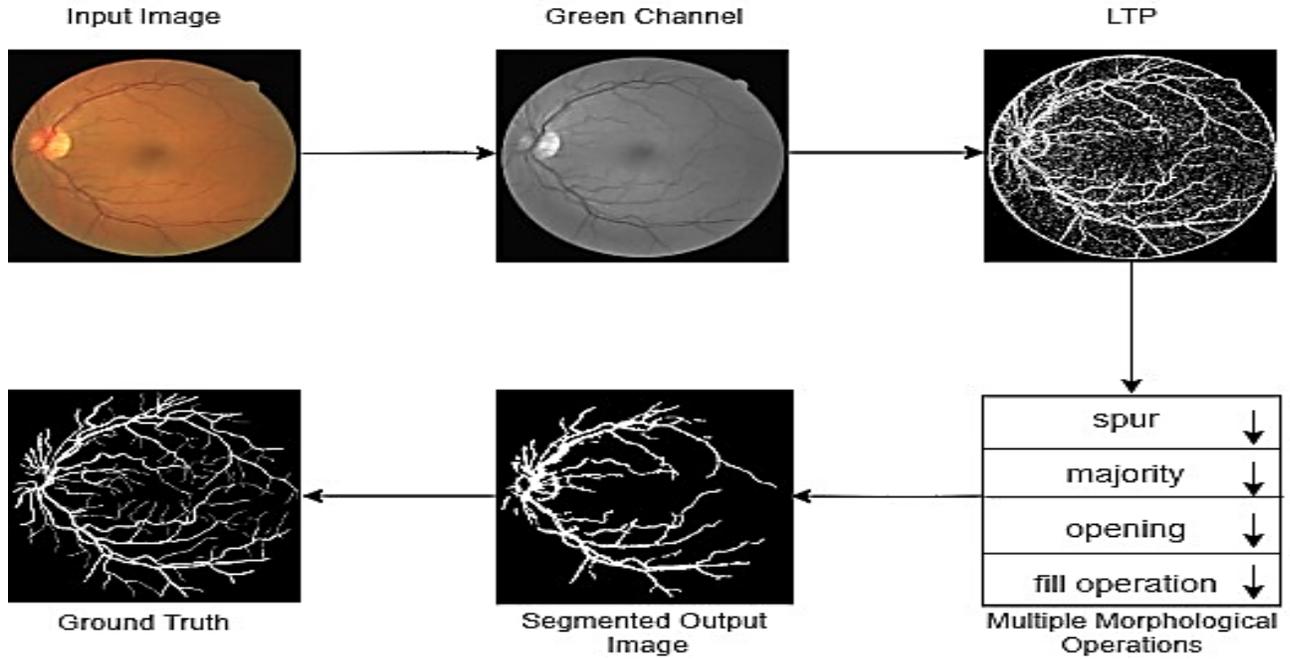


FIGURE 2: FLOW OF THE PROPOSED LTP BASED VESSEL SEGMENTATION ALGORITHM

a) SPUR OPERATION

This operation is used 100 times and removes the spur pixels as shown below.

$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{matrix}$	becomes	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{matrix}$
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b) MAJORITY OPERATION

This operation is used 10 times and it sets value 1 to the pixel if there are five or more 1 pixels in its 3×3 neighbourhood, otherwise pixels value is set 0.

c) OPENING OPERATION

Performs morphological opening that is erosion followed by dilation. The morphological filters result in top-hat image. Opening operator (\circ) on image (I) and (S_o) structuring element is described in Eq (3). This operation is performed 50 times.

$$Topen = I \circ S_o \quad (3)$$

d) FILL OPERATION

Fill operation isolates interior pixels that is individual 0s that are surrounded by 1s, such as in this pattern centre pixel as shown below.

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix}$$

In our work, the fill operation is used 10 times and finally the segmented output image is obtained.

IV. SIMULATION RESULTS

We simulate the developed LTP based algorithm on intel-core i7 machine that has 8GB of RAM along with MATLAB 2016.

Datasets: we investigate two publicly available DRIVE [11] and the STARE [7] datasets. To evaluate and compare our algorithm, we consider three parameters that are Accuracy (Acc), Sensitivity (Se), and Specificity (Sp) as defined below.

$$Acc = \frac{(TP+TN)}{(TP+FP+TN+FN)} \quad (4)$$

$$Se = \frac{(TP)}{(TP+FN)} \quad (5)$$

$$Sp = \frac{(TN)}{(TN+FP)} \quad (6)$$

Where TP denotes True Positives, TN as True Negatives, FP shows False Positives, and FN as False Negatives. Below, we discuss our results.

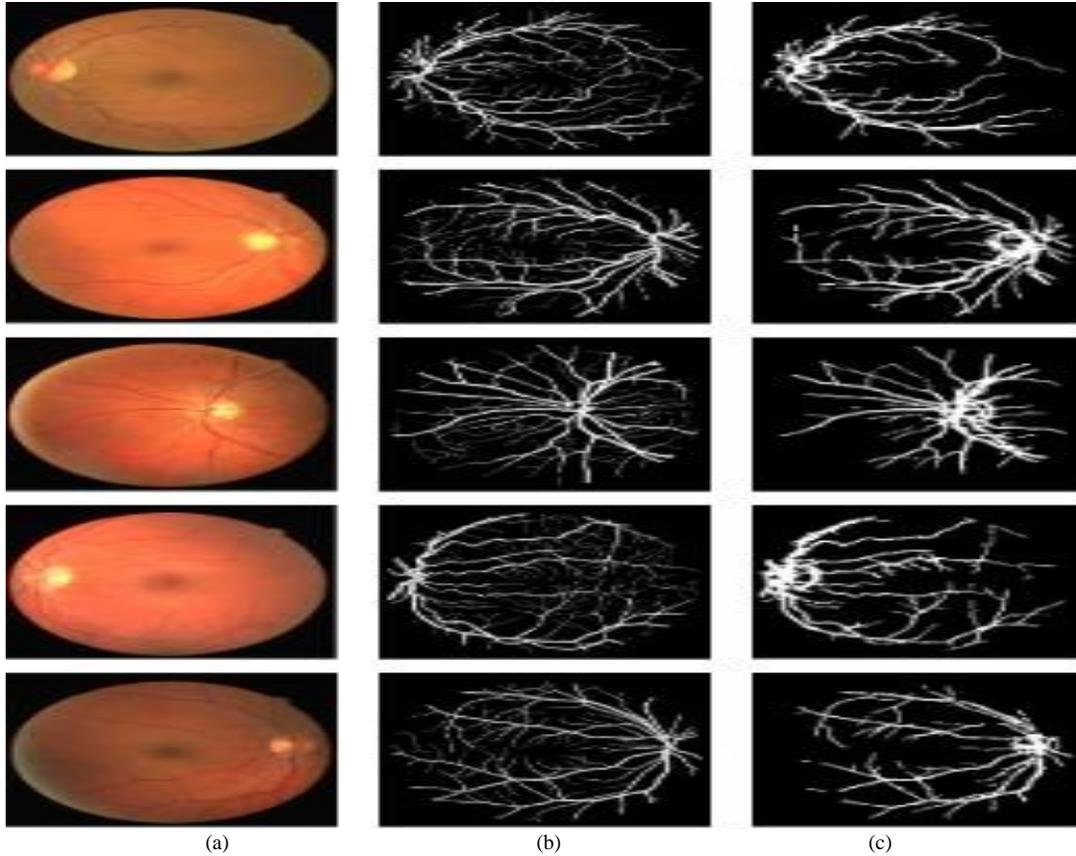


FIGURE 3: DRIVE DATASET: (A) INPUT IMAGE, (B) GROUND TRUTH, AND (C) OUTPUT IMAGE

A. SEGMENTATION ANALYSIS

The segmented output of our proposed method along with its ground truth on few images from both datasets is shown in FIGURE. 3 and FIGURE 4, respectively with following observations.

- As shown in FIGURE 3 and FIGURE 4, the obtained output images are close to ground truth. As shown in row 3rd of FIGURE 3 and 4th row of FIGURE 4 that thick vessels are detected accurately. Similarly, on top row of FIGURE 3 and 5th row of FIGURE 4 that both the thick and thin vessels are segmented close to its relevant ground truth.
- On Drive dataset, as shown in FIGURE 3, we observe that our proposed method accurately segments the thick vessels in the exact direction as specified by its ground truth. Importantly, on this dataset, most the fine vessels are also precisely segmented and located. This particular fact can be observed in 4th row of FIGURE 3.
- Similarly, on the Drive dataset, as shown in 5th and 2nd rows of FIGURE 3, that our proposed method exactly segments the small branches that coincide with the thin vessels.
- As shown in 1st, 2nd and 3rd rows in FIGURE 4 and on STARE database, the proposed LTP based algorithm segments thick vessels precisely. In this case, the segmented image yielded by our developed algorithm has a close resemblance to its ground truth. Some false positive are observed in the output.

- Image of our proposed algorithm, however that does not disturb vessels appearance.

B. COMPARISON

The proposed LTP based algorithm is compared with few recently developed algorithms as shown in Table I with following important observations.

- As shown in Table I and on the DRIVE dataset, the proposed method outperforms [21] and [23] in terms of Acc and ranks 3rd in supervised and 5th in unsupervised methods.
- In particular, [24] uses the deep learning model to achieve higher Acc at the cost of much high complexity. On the STARE dataset, the proposed method ranks 7th in Acc comparison. In this case supervised methods yield slightly higher Acc as they use higher resources, such as more training data [27]–[37].

C. DISCUSSION

For supervised methods shown in TABLE I, on the DRIVE dataset, the proposed method yields higher Acc than [23]. Similarly, as shown in FIGURE 3 and FIGURE 4 that some false positives are present in the interior of output images of our proposed algorithm. However, it does not distress the orientation of the vessels indicated by ground truths. Generally,

the proposed LTP based segmentation algorithm revealed that it detected the thick vessels with comparable *Acc.*

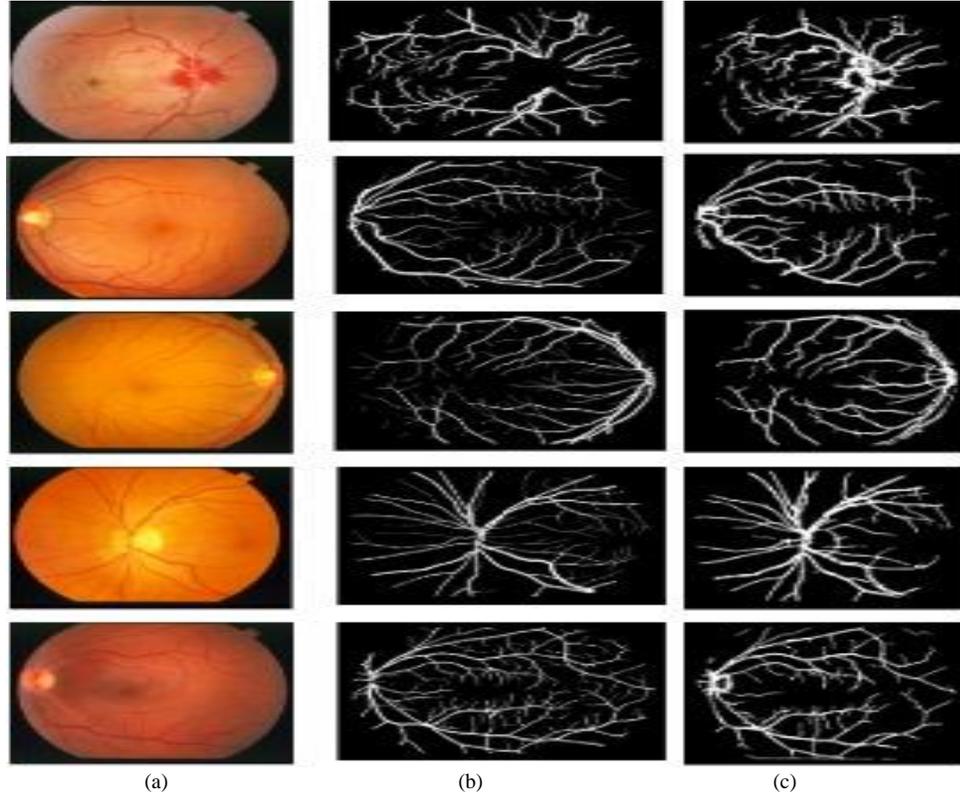


FIGURE 4: STARE DATASET: (A). INPUT IMAGE (B). GROUND TRUTH, AND (C). OUTPUT IMAGE

TABLE I
PERFORMANCE COMPARISON

Ref	Method	DRIVE			STARE		
		Accuracy (<i>Acc</i>)	Sensitivity (<i>Se</i>)	Specificity (<i>Sp</i>)	Accuracy (<i>Acc</i>)	Sensitivity (<i>Se</i>)	Specificity (<i>Sp</i>)
[23]	Supervised	0.9421	0.7560	0.9696	0.9477	0.7202	0.9733
[24]		0.9271	0.9382	0.9255	0.9378	0.9598	0.9352
[25]		0.9476	0.7743	0.9725	0.9554	0.7791	0.9758
[1]		0.9470	0.7421	–	0.9472	0.8004	–
[15]		0.9450	0.7390	0.9740	0.9400	0.7370	0.9620
[17]	Unsupervised	0.9447	0.7528	–	–	–	–
[21]		0.9411	–	–	0.9489	–	–
[22]		0.9450	0.7213	0.9665	0.8876	0.7550	0.9038
Proposed		0.9435	0.6114	0.9754	0.9341	0.5011	0.9684

Whereas, in case of fine vessels the proposed algorithm had lower *Se*, which resulted in failure to segment fine vessels. On the DRIVE dataset, our proposed LTP based segmentation algorithm yields 0.9435, 0.6114, and 0.9754 *Acc*, *Se*, and *Sp* values, respectively. Whereas, on the STARE dataset, our method produces 0.9341, 0.5011, and 0.9684 *Acc*, *Se*, and *Sp* values.

V. CONCLUSIONS AND FUTURE WORK

Accurate vessels segmentation is a prerequisite step in ophthalmology to diagnose fatal diseases. This paper discusses a robust segmentation algorithm. The proposed method processes only the Green channel from the input image. On Green channel, Local Ternary Patterns (LTP) is applied. To remove

noise from the small vessels multiple morphological operations such as spur, majority, opening and fill operations are applied to get the segmented image. The proposed LTP based vessel segmentation algorithm outperforms newly reported methods in terms of Accuracy on DRIVE and STARE datasets. In the future, the proposed algorithm can be further investigated to handle pathological cases through robust methods, such as principal component analysis and contrast enhancement.

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