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Giving Concept of an Analog Notch Filter for Removing Interference from Engineering Systems to Engineering Students

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Abstract—**Power lines exist everywhere. Interference of power frequency with signals of other systems (telephone, music, and so on) is inevitable. This results in deteriorating the quality signals of other systems to be used for accomplishing required tasks. In this paper, a continuous-time notch filter is employed for removing the interference or annoying hums caused by the powerline frequency or its second or third harmonics in two engineering systems, namely the telephone transmission system, and the tape deck system. The notch filter accomplishes this task by attenuating the magnitude of the interference signal at a notch frequency to zero (ideally speaking). The paper may serve as a starting point for the engineering students who want to learn the basics of analog filters. MATLAB/Simulink based simulation results display that the notch filter can successfully filter out power frequency interference/hums or its harmonics to get the required quality signal at the output.**

Index Terms— **Notch filter, telephone transmission system, tape deck, powerline frequency, harmonics.**

I. Introduction

The quality of the signals of various engineering systems deteriorates due to the interference of the powerline frequency (single-frequency interference) or its harmonics (multiple-frequencies interference). These frequency components need to be removed. One possible solution is the utilization of a notch filter to remove 50-Hz (the powerline frequency in most parts of the world, Europe, and Pakistan) or its harmonics. Conceptually speaking, notch filter removes the frequency components by attenuating their magnitudes at the required notch frequencies to a maximum without disturbing the other useful frequency components (the original information). In this paper, an analog notch filter is applied to two engineering applications for removing hums occurring at powerline frequency.

Many researchers have successfully employed a notch filter, analog as well as digital, for eliminating interference embedded with original information of the systems. A digital FIR equiripple notch filter [1], and an adaptive filter and a notch filter [2] were successfully applied for eliminating powerline interference from Electrocardiogram (ECG) signal. In [3], powerline interference cancellation was carried out by a linear Kalman notch filter. Reference [4] suggested a computationally effective solution in the form of two-pole and multi-pole notch filters for Global Navigation Satellite System (GNSS) interference detection and mitigation. Reference [5] suggested the utilization of multiple-notch filtering methods for notching harmonic powerline interference with improved transient behavior.

Other than notch filters, other filtering mechanisms are also reported in the literature. Reference [6] employed S-transform as a filter in filtering out powerline interference from biomedical signals. In [7], the ECG signal was de-noised from a 50-Hz Powerline Interference (PLI) noise by Discrete Wavelet Transform (DWT). Suppression of powerline noise from the ECG signal was also carried out using an FIR digital filter implemented with a hamming window [8] and a rectangular window [9].

In this paper, two engineering applications are considered to highlight the significance of a notch filter for removing the powerline frequency or its harmonics. Very limited research work is found in the literature where a notch filter is used for filtering out interference for the mentioned engineering systems. The other aim of presenting the paper is to make engineering students understand the basics of a notch filter and then to apply it to real engineering applications to remove powerline interference and noise.

The paper is organized in the following way. Section II describes the fundamentals of a notch filter. The considered systems, i.e., telephone transmission system, and the audio deck system are detailed in Section III along with the simulation results to validate the effectiveness of the notch filter in removing the interference. In the end, in Section IV, conclusions are deduced.

II. Analog Notch Filter Fundamentals

A notch filter is essentially a stop-band or stop-elimination filter that stops the notch frequencies and allows all other frequencies to pass. The circuit diagram of a typical notch filter is shown in Fig. 1. It consists of a capacitor and an inductor connected in parallel along with a resistor.

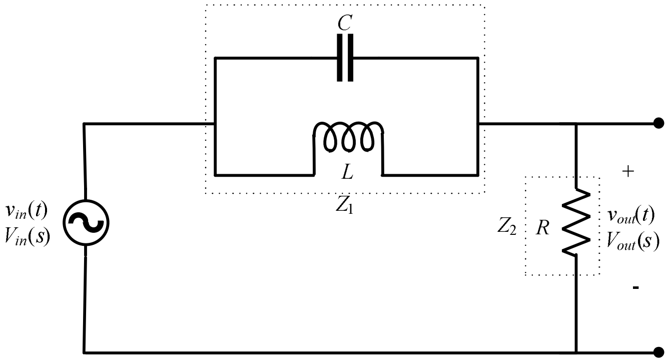

Fig. 1. The circuit diagram of a notch filter.

To get more insight into the functionality of a notch filter, we derive the transfer function of it as

$$G(s)=\frac{V_{out}}{V_{in}}=\frac{Z_2}{Z_1+Z_2}=\frac{R}{\dfrac{Ls}{s^2LC+1}+R}$$
$$=\frac{R\left(s^2LC+1\right)}{Ls+s^2LCR+R}=\frac{s^2RLC+R}{LCRs^2+Ls+R} \tag{1}$$
$$G(s)=\frac{s^2+\dfrac{1}{LC}}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}$$

with

$$Z_1=\frac{(sL)\left(\dfrac{1}{sC}\right)}{sL+\dfrac{1}{sC}}=\frac{Ls}{s^2LC+1}$$

$$Z_2=R$$

Replacing $s=j\omega$ gives

$$G(j\omega)=\frac{V_0}{V_{in}}=\frac{(j\omega)^2+\dfrac{1}{LC}}{(j\omega)^2+\left(\dfrac{j\omega}{RC}\right)+\dfrac{1}{LC}} \tag{2}$$

The notch filter is essentially a two-zero two-pole system. The bode diagram (the magnitude plot) is roughly sketched in Fig. 2.

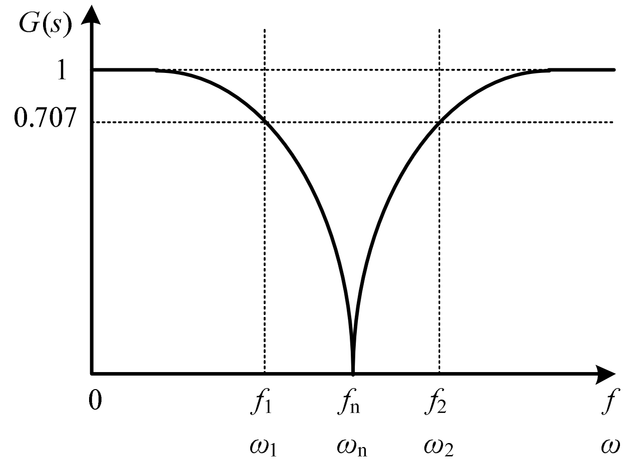

Fig. 2. Bode plot (magnitude curve) of a notch filter.

The frequencies ω_1, ω_2 and ω_n are geometrically interrelated by $\omega_1\omega_2=\omega_n^2$, whereas bandwidth BW of the filter is $\omega_1-\omega_2$. At the notch frequency $\omega_n=1/\sqrt{LC}$, the transfer function $G(s)$ reduces to zero, i.e., $|G(j\omega_n)|=0$. The magnitude curve of the Bode plot suggests that there exists a symmetry which is expressed by

$$\begin{cases}|G(j\omega_1)|=|G(j\omega_2)|\\ \theta(j\omega_1)=-\theta(j\omega_2)\end{cases} \tag{3}$$

It should be noted that the minimum the notch width is, the better the notch filter is characteristically. The filter with less notch width eliminates typically the required frequency components without reducing the magnitude of other frequency components.

III. Design Examples

In this section, the application of a notch filter to two systems, i.e., telephone transmission system and tape deck system is presented for eliminating 50-Hz interference or second or third harmonics of it.

A. A Telephone Transmission System

A telephone transmission system suffering from a 50-Hz power frequency along with a notch filter is shown in Fig. 3. Let the equivalent resistance of the telephone system be denoted by R_{eq}. The purpose of the notch filter is to filter out 50-Hz interference.

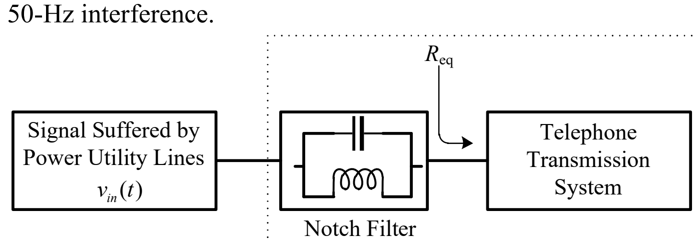

Fig. 3. A telephone transmission system suffering from a 50-Hz power frequency along with a notch filter.

By the inspection of Fig. 3, the equivalent circuit model for the telephone transmission system can be derived. It is the same circuit, as indicated in Fig. 1. It requires only the replacement of R with R_{eq}. As a result, the transfer function of the system (LC network followed by R_{eq}) is given by

$$G(j\omega) = \frac{V_{out}}{V_{in}} = \frac{(j\omega)^2 + \frac{1}{LC}}{(j\omega)^2 + \left(\frac{1}{R_{eq}C}\right) j\omega + \frac{1}{LC}} \quad (4)$$

For notching $\omega_n = 1/\sqrt{LC} = 2\pi(50) = 100\pi$, if we select an easily available capacitor with a standard value of 150 μF, then the value of an inductor comes out to be 0.0675 H.

The input signal to the notch filter $v_{in}(t)$ contains a sinusoid of 1000-Hz (the original telephone signal to be passed) and a sinusoid of 50-Hz (the power frequency to be stopped) and is given mathematically by

$$v_{in}(t) = \underbrace{1\sin\left[(2\pi)50t\right]}_{signal\ to\ be\ stopped} + \underbrace{0.5\sin\left[(2\pi)1000t\right]}_{signal\ to\ be\ passed} \quad (5)$$

For the sake of simplification, the magnitude of the voltage signal whether it is the original signal to be sent or the interfering powerline signal is kept unity or fraction of it, as we are more interested in notching power frequency or harmonics of it.

From the simulation result shown in Fig. 4, one can observe that $v_{out}(t)$ does not contain 50-Hz interference.

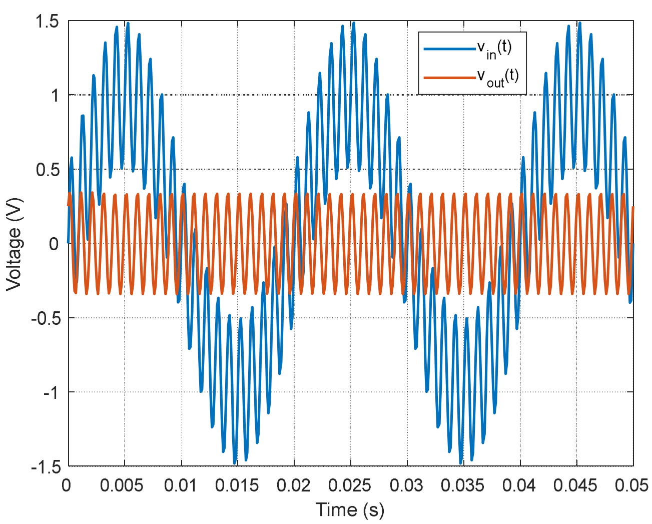

Fig. 4. The output voltage without the 50-Hz interference.

If we include the second- and third-harmonic junk in $v_{in}(t)$, then $v_{in}(t)$ takes the following form

$$v_{in}(t) = \underbrace{\begin{matrix} 1\sin\left[(2\pi)50t\right] + 0.3\sin\left[(2\pi)100t\right] \\ +0.2\sin\left[(2\pi)150t\right] \end{matrix}}_{signals\ to\ be\ stopped} + \underbrace{0.5\sin\left[(2\pi)1000t\right]}_{signal\ to\ be\ passed} \quad (6)$$

As can be observed from Fig. 5, the same notch filter used for removing 50-Hz interference can remove second and third harmonics as well.

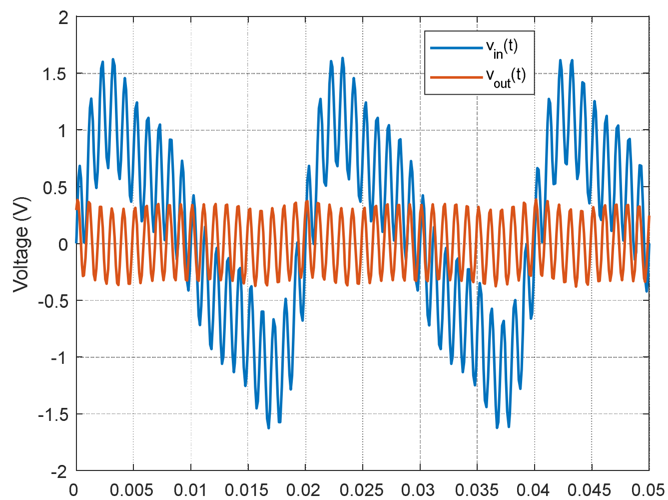

Fig. 5. Input and output voltage waveforms.

B. A Cassette Deck System

Another system, i.e., a cassette deck system, also undergoes the effect of powerline frequency of 50-Hz. Remember that Audio Frequency Spectrum (a range of audible frequencies) spans from 20 Hz to 20 kHz. A cassette deck system, if improved well, could record and play any frequency lying within Audio Frequency Spectrum. However, cheaper decks usually come without filters like MPX (Multiplex Signal) and thus are not able to remove noise to ensure better sound quality. The paper proposes a notch filter for filtering out such noise.

A cassette deck system (block diagram and circuit diagram) suffering from 50-Hz power frequency along with a notch filter is shown in Fig. 6. Let the equivalent resistance of the cassette tape player and the power amplifier with the speaker be denoted by R_{deck} and R_{amp} respectively. Their values are taken 50 Ω and 1 kΩ respectively. $v_{out}(t)$ is essentially $v_{amp}(t)$.

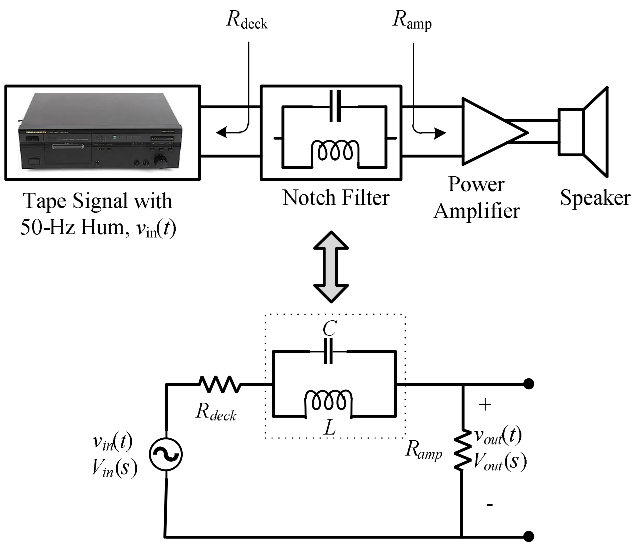

Fig. 6. A cassette deck system (block diagram and circuit diagram).

Application of voltage division rule (VDR) to the circuit shown in Fig. 6 results in the following transfer function

$$G(s)=\frac{V_{out}}{V_{in}}=\frac{R_{amp}}{R_{deck}+R_{amp}+\left(sL\parallel\frac{1}{sC}\right)} \tag{7}$$

Manipulation of (7) gives

$$G(s)=\frac{V_{out}}{V_{in}}=\frac{R_{amp}}{R_{deck}+R_{amp}}\left[\frac{LCs^2+1}{LCs^2+\left(\frac{L}{R_{deck}+R_{amp}}\right)s+1}\right] \tag{8}$$

Replacing $s=j\omega$ gives

$$G(j\omega)=\frac{V_{out}}{V_{in}}=\frac{R_{amp}}{R_{deck}+R_{amp}}$$

$$\left[\frac{(j\omega)^2LC+1}{(j\omega)^2LC+\left(\frac{L}{R_{deck}+R_{amp}}\right)(j\omega)+1}\right] \tag{9}$$

For notching $\omega_n=1/\sqrt{LC}=2\pi(50)=100\pi$, if we select an easily available capacitor with a standard value of 22 µF, then the value of an inductor comes out to be 0.4606 H.

The input signal to the notch filter $v_{in}(t)$ contains a sinusoid of 2000-Hz (the original music signal to be passed) and a sinusoid of 50-Hz (the power frequency to be stopped) and is given mathematically by

$$v_{in}(t)=\underbrace{1\sin\left[(2\pi)50t\right]}_{\text{signal to be stopped}}+\underbrace{0.5\sin\left[(2\pi)2000t\right]}_{\text{signal to be passed}} \tag{10}$$

From the simulation result shown in Fig. 7, one can observe that $v_{out}(t)$ does not contain 50-Hz interference.

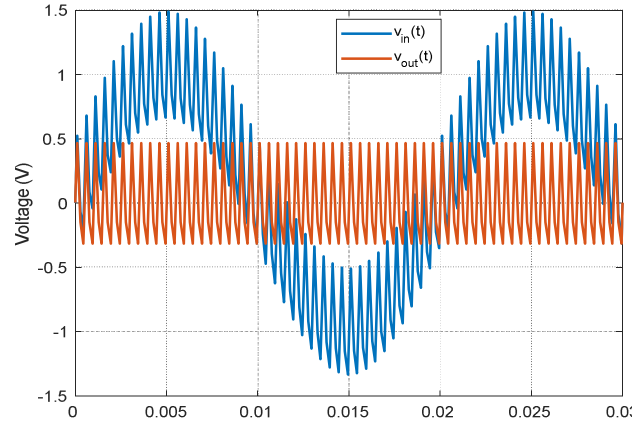

Fig. 7. 50-Hz free output voltage along with the input voltage.

To check whether the single-stage filter can eliminate harmonics of powerline frequency, we include the second- and third-harmonic junk in $v_{in}(t)$. $v_{in}(t)$ then takes the following form

$$v_{in}(t)=\underbrace{\begin{array}{l}1\sin\left[(2\pi)50t\right]+0.3\sin\left[(2\pi)100t\right]\\+0.2\sin\left[(2\pi)150t\right]\end{array}}_{\text{signals to be stopped}}\;\underbrace{+0.5\sin\left[(2\pi)2000t\right]}_{\text{signal to be passed}} \tag{11}$$

As can be observed from Fig. 8, although a single notch filter can eliminate 50-Hz interference efficiently, it finds it difficult to remove 100-Hz (second harmonics) and 150-Hz (third harmonics). The author suggests that these harmonics can be eliminated by cascading two more notch filters in series. The circuit diagram where three notch filters are connected in series to remove 50-Hz, 100-Hz, and 150-Hz hums, is shown in Fig. 9. For the case of notching 100 Hz and 150 Hz, we take the same capacitor of a value of 22 µF as in the 50 Hz case. Correspondingly, the values of inductors are calculated to be 0.2533 H and 0.1126 H, respectively. It can be seen from Fig. 10 that the cascaded notch filter successfully filters out harmonics of the power frequency.

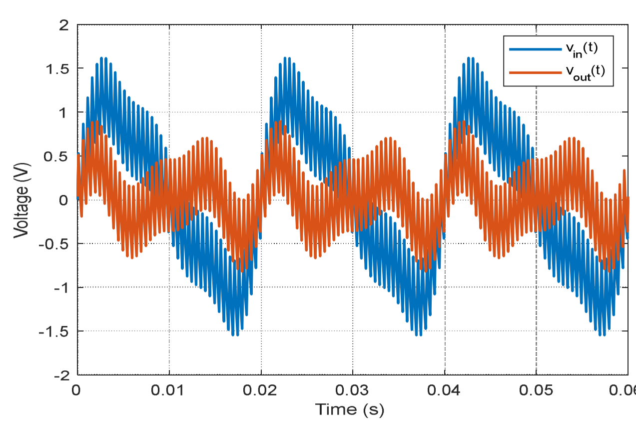

Fig. 8. Distorted output voltage along with input voltage.

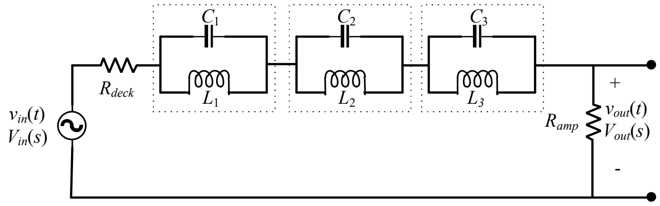

Fig. 9. Three notch filters are connected in series to remove 50-Hz, 100-Hz, and 150-Hz hums.

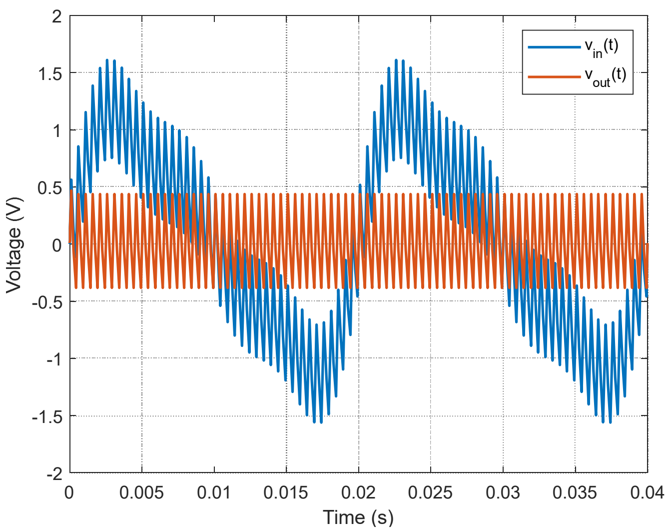

Fig. 10. Harmonics free output voltage along with the input voltage.

IV. Conclusion

In this paper, unwanted signals in the form of powerline frequency, i.e., 50-Hz, or its second or third harmonics that get interfered with the telephone transmission system and deck system, are removed with the help of a notch filter. It reinforces the established fact that notch filter can easily notch the unwanted frequencies. To notch second and third harmonics of power frequency, a cascaded notch filter may have to be used. The paper may come for the rescue of engineering students who intend to understand the basics of analog notch filters. The development of hardware for the considered systems can be carried out in the future.

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