

Modeling exchange rate volatility with GARCH models: A comparison based on a volatility breaks

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ABSTRACT

Volatility has been defined as a worthy indicator of uncertainty, which has implications on various factors such as international trade, investment decisions, and valuation for a currency. This paper investigates the structure of volatility in the exchange rate data by considering a structural break. Monthly data of Pakistani Rupee's exchange rate is considered for 21 years starting from January 2000 to November 2020. The State Bank of Pakistan supplied the nominal exchange rate data. It is found that the threshold GARCH (TGARCH) model is more suitable to estimate the volatility of the exchange rate for comprehensive data of 21 years. Results show that if data is bifurcated based on structural break, then the low and high volatility can be estimated more accurately with exponential GARCH (EGARCH) and square GARCH (SGARCH), respectively. Before the structural break, the exchange rate volatility is lower than that after the structural break due to clustering volatility. Also, the research showed that the volatility clustering effect is found in the volatility of exchange rate data as low volatility is followed by low and high volatility is followed by high volatility for a prolonged period.

Keywords: Volatility, exchange rate, structural break, GARCH, TGARCH, SGARCH, EGARCH.

INTRODUCTION

In this era of modernization, an exchange is considered one of the key elements in an economy. It is connected to the inflation rate, interest rate, government debts, and speculation. So, it is imperative to study the volatility of the exchange rate to help academics and policymakers draw sound economic decisions. International trade and economic welfare are likely to depend upon the exchange rate volatility (Asseery & Peel, 1991). Hence, it is necessary to evaluate and fathom the exchange rate volatility behaviour to draw a reasonable monetary policy for a country (Longmore & Robinson,

2004). Hence, all the stakeholders, including policymakers, researchers, and businessperson, are very keen to understand the volatility behaviour of the exchange rate that can help them to draw such decisions that can minimize the adverse effects of volatility of exchange rate for an economy (Abdullah et al., 2017).

For developing counties, evidence suggests that economic agents hold a vital proportion of their wealth in foreign currencies such as dollars (Tule et al., 2014; Udoh & Udeaja, 2019). The generalized autoregressive conditional heteroscedasticity (GARCH) model is widely used to capture time-series volatility (Agnolucci, 2009; Angabini & Wasiuzzama, 2011). However, this model is unable to describe some essential features of asset return (Cai & Li, 2019). To overcome this limitation, researchers have established some adapted GARCH models. Nelson (1991) presented the exponential GARCH (EGARCH) model, and Glosten et al. (1993) introduced the GJR-GARCH model. GJR-GARCH and EGARCH models' purpose is to elaborate on the positive and negative return impact on conditional volatility. Then, Zakoian (1994) presented the threshold GARCH (TGARCH) model for the same purposes (Hongwiengjan & Thongtha, 2020).

To understand the time-series volatility dynamics, the GARCH model is considered one of the paramount techniques to understand and measure various economic elements such as stock exchange, interest rate, and exchange rate. Although this technique cannot elaborate on some features of assets (Cai & Li, 2019). Hence, various modifications are made in GARCH models to make them useful. Univariate GARCH models contain threshold (TGARCH), Glosten Jagannathan Runkle (GJR-GARCH), exponential (EGARCH), and square (SGARCH). In contrast, multivariate GARCH models include DCC-GARCH, CCC-GARCH, and VCC-GARCH, to make it more convenient for univariate and multivariate time series (Hongwiengjan & Thongtha, 2020). For comparison, several GARCH models were predicted based on with and without volatility breaks. It was evident that various models of GARCH rejected the presence of leverage except for those with breaks in volatility. It is, therefore, concluded that the results of the analysis were improved when volatility breaks are considered in the model that suggested the significant improvements in the suggested GARCH models. To estimate the exchange rate volatility, Clement and Samuel (2011) also examined the monthly exchange rate data for four years starting from 2007. In this research, an empirical estimation for the exchange rate (USD/PKR) via EGARCH, SGARCH, and TGARCH model is evaluated and compared to a structural break in the monthly data. The results are diverse for both segmented periods, and hence, the outcomes are compared within a time series of the exchange rate.

In section 2, a piece of literature review about the exchange rate, EGARCH, EGARCH, and TGARCH model is provided. An approximation of the exchange rate volatility is fathomed in section 3 and a computational part of the analysis and their results are interpreted further in section 4. This part is followed by the discussion and conclusion section, respectively.

RESEARCH OBJECTIVES

The TGARCH model is considered a more accurate technique to predict volatility (Munir & Ching, 2019). The objective of the study is twofold. Firstly, to estimate the exchange rate volatility using GARCH models. Secondly, to elaborate on the role of volatility break in the monthly dataset of the exchange rate, how the impact of structural break may affect the modeling of the currency's unpredictability.

Research Question

What is the volatility break role by using GARCH combinations to estimate the exchange rate uncertainty?

LITERATURE REVIEW

The GARCH model was presented by Bollerslev (1986), and it is used to predict non-constant volatility that depends upon time. It is cleared that the GARCH model presents healthier results for persistent and smooth change volatility (Chen et al., 2013). For such reasons, the application of the GARCH model is widely used in various time series models and sectors (Agnolucci, 2009; Angabini & Wasiuzzama, 2011), although this model is unable to describe some features of asset return (Cai & Li, 2019). To overcome this limitation, researchers have established some adapted GARCH models. Nelson (1991) presented the exponential GARCH (EGARCH) model, and Glosten et al. (1993) introduced the GJR-GARCH model. The GJR-GARCH and EGARCH models' purpose is to elaborate on the positive and negative return impact on conditional volatility. Then, Zakoian (1994) presented the TGARCH model for the same purposes (Hongwiengjan & Thongtha, 2020).

Estimating the exchange rate volatility is of paramount importance because of its implications in various sectors. Bala and Asemota (2013) observed the volatility of the exchange rate by using GARCH models. They examined the monthly exchange rate return of Nigerian currency to the US dollar (\$), British pound, and the euro. For comparison, several GARCH models were predicted based on with and without volatility breaks. It was

evident that various models of GARCH rejected the presence of leverage except for those with breaks in volatility. It is, therefore, concluded that the results of the analysis were improved when volatility breaks are considered in the model that suggested the significant improvements in the suggested GARCH models. To estimate the exchange rate volatility, Clement and Samuel (2011) also examined the monthly exchange rate data for 4 years starting from 2007. Their results revealed that when the exchange rate series is non-stationary, then the series's residuals are asymmetric. The results found that as the return volatility is considered persistent with time, this effect is beneficial for policymakers in the government to understand and organize exchange rates (Abdullah et al., 2017).

Çağlayan and Dayıoğlu (2013) analyzed the exchange rate and established the model of exchange rate volatility for four countries (Mexico, Indonesia, South Korea, and Turkey) to US dollar (\$) using various GARCH models. They examined monthly exchange rate data for 20 years opening from 1993 to 2013 to explore fat-tailed features and leverage effects. They acknowledged leveraging effects and asymmetric behaviour of said countries' data for the exchange rate to the US dollar.

Herwartz and Reimers (2002) studied the daily exchange rate volatility between the US dollar & the Deutsche mark (DM) and the Japanese yen (JPY). Deutsche mark (DM) was taken for a period of 24 years ranging from 1975 to 1998. Their study used GARCH (1, 1) model along with leptokurtic to understand and capture the volatility clustering. Results showed evidence that structural change elements were subject to change to monetary policies in Japan and the US. Vee et al. (2011) also inspected the accuracy in the prediction of GARCH (1, 1). They used daily data on the exchange rate of the Mauritian Rupee against the US dollar. They likened the root mean squared error (RMSE) and mean absolute error (MAE) of the said models depending upon an estimation of forecasts. Their results revealed that the GARCH model with GEC showed better results for accuracy in estimation than Students' t-distribution (Abdullah et al., 2017).

Following Zakoian (1994) work, researchers have worked on the TGARCH model and applied this technique in different underlying asset prices to estimate and predict volatility such as carbon, crude oil, sugar (Hasan et al., 2013; Godana et al., 2014). The TGARCH model is also examined to elaborate on the mortgage risk price of houses and capture the house price volatility (Lee et al., 2015; Hongwiengjan & Thongtha, 2020).

TGARCH model is considered a more accurate technique to predict volatility (Munir, & Ching, 2019). The TGARCH model is used to elaborate the mortgage risk price of houses and to capture the house price volatility accurately (Lee et al., 2015; Hongwiengjan & Thongtha, 2020). Empirical results showed that TGARCH is one of the best models to estimate the

volatility of Bitcoin (Gyamerah, 2019). To the best of my knowledge, literature is silent to express the exchange rate volatility with GARCH model combinations. Besides, to compare and contrast the exchange rate volatility, the structural break would be another vital addition to the literature. Therefore, finding a suitable GARCH model and an applicable estimation of the variance would be an honoured contribution in the study of exchange rate volatility.

METHODOLOGY

Data and variable construction

For this study, the monthly data of the Pakistani Rupee exchange rate against the US dollar is taken for 21 years, starting from January 2000 to November 2020. The State Bank of Pakistan supplied the nominal exchange rate data. Since the data was non-stationary (shown in Figure 3), it was transformed to become stationary by using the following formula to calculate the exchange rate.

$$r_t = \ln \left(\frac{fx_t}{fx_{t-1}} \right) \quad (i)$$

$$\text{or, } r_t = \ln(fx_t) - \ln(fx_{t-1}) \quad (ii)$$

Here, r_t stands for exchange rate return at period t ; fx_t and fx_{t-1} stand for the nominal exchange rate of the USD/PKR at period t and $(t-1)$, respectively. The statistical software Stata is used for the application of GARCH models.

Specification of different models

It is evident that for applying the GARCH family models, estimation, and modelling of volatility, the correct mean equation is vital. On the other hand, if the study fails to choose the correct mean equation for its modelling and address the issue of autocorrelation, then the work may not achieve its desired results in volatility estimation (shown in Figure 4). Hence, to overcome this problem, various GARCH model combinations are examined to specify the model correctly. Mean Equations:

$$r_t = \mu + \varepsilon_t \quad (iii)$$

$$r_t = \mu + \rho_1 r_{t-1} + \varepsilon_t \quad (iv)$$

$$r_t = \mu + \rho_1 r_{t-1} + \rho_2 r_{t-2} + \varepsilon_t \quad (v)$$

This study used five diverse GARCH models to specify the variance equations to estimate volatility in the exchange rate. The GARCH model presents healthier results for persistent and smooth change volatility (Chen et al., 2013). This research estimated the variances using ARCH, GARCH,

TGARCH, EGARCH, and SGARCH models. A change examined the sensitivity of predicted models in the distribution in assumptions.

Estimation Results and Discussion

Table 1 displays a descriptive analysis of the monthly exchange rate for 21 years. The mean value of the exchange rate during the given period is 87.39, while the Standard deviation is 29.9. Log ER and Devalue are transforming the exchange rate in logarithm and the exchange rate difference, respectively. Both Log ER and Devalue show lessor values than Ex. Rate. The mean value of Log ER and Devalue are 4.42 and 0.01, respectively. Besides, the standard deviation and range of Log ER are more significant than the Devalue.

Table 1. Descriptive Analysis

Variable	Obs.	Mean	SD	Min	Max
Ex. Rate	251	87.39	29.91	51.88	167.28
Log ER	251	4.42	0.32	3.95	5.12
Devalue	250	0.01	0.02	-0.07	0.10

Note: From January 2000 to November 2020

Figure 1 represents the raw monthly exchange rate (USD/PKR) from January 2000 to November 2020. Both of the graphs in Figure 1 are depicting the same data but with different angles. It is clear from the figure that there is a structural break in the data in December 2016, represented by the red line shown in the graph. The structural break is checked with the Zandrews test and the results, as shown below. Minimum t-statistic -2.988 at 2016m12 (Obs. No. 204). Hence, the raw monthly exchange rate data is divided into two parts based on the structural break test. The first part is starting from an initial point, January 2000 to December 2016, and similarly, the second part is starting from December 2016 to the end of the dataset. Bala and Asemota (2013) concluded that the analysis results were improved when volatility breaks are considered in the model by using the GARCH models. Before the structural break, the exchange rate volatility is lower than that after the structural break due to clustering volatility. Low volatility is followed by low volatility, and high volatility is followed by high volatility for a prolonged period. This phenomenon is due to a change in government regime, an increase in government debts, and Pakistan's inflation rate.

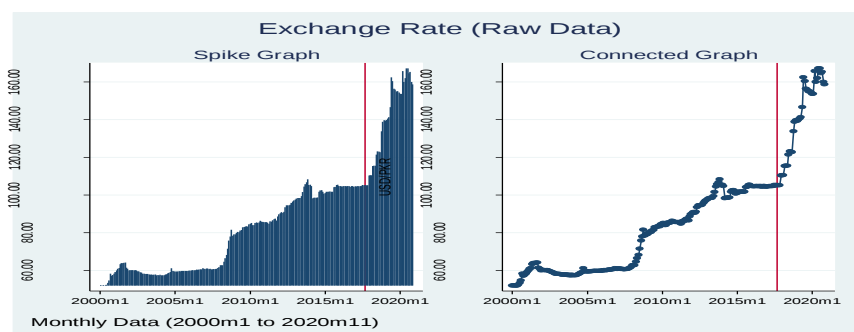


Figure 1. Exchange rate (raw data)

Figure 2 represents that there is no serial correlation in the data. As the p-value of the white noise test is 0.65, shown in Figure 2, which is insignificant and fails to reject the null hypothesis. Frequency is taken on the x-axis, while the cumulative periodogram is taken on the y-axis. And the null hypothesis stated that there is no serial correlation in the given data. The figure represents that the residuals are within the boundaries and limits. Hence, it is justifying that there is no serial correlation in the exchange rate's monthly data.

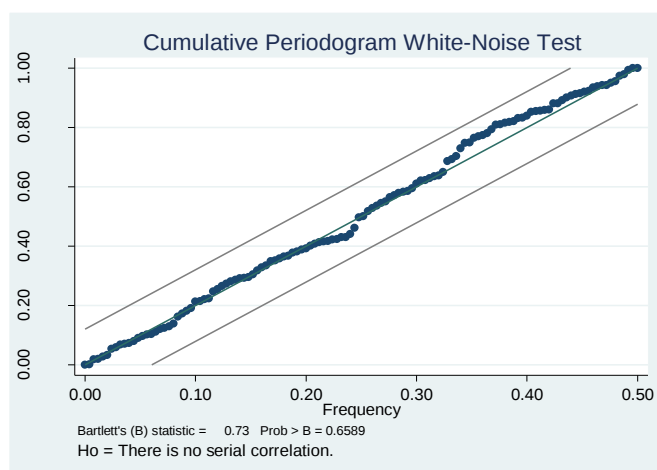


Figure 2. White-Noise test

Table 2 shows the GARCH modelling applied to the data before the structural break (volatility). The range of this dataset is from January 2000 to December 2016. The table represents the various models, including ARCH, GARCH, TGARCH, EGARCH, and SGARCH and the results showed that all of the ARCH/GARCH models are significant at a 1% significance level. Besides, the coefficients for all the models are positive. GARCH model presents healthier results for persistent and smooth change volatility (Chen et al., 2013). The selection criterion is Akaike's and Bayesian information criterion (AIC and BIS), as suggested by Abdullah et al. (2017), which states that the lowest value is used to select an appropriate GARCH model. ARCH

model has more AIC and BIS value than GARCH models. This research further bifurcated the GARCH family, and it is shown that these models have very competitive value. Based on AIC and BIS, the EGARCH model is found appropriate to measure the exchange rate volatility. Furthermore, e GARCH has the highest z-value, which is 69.37, and its standard error is the lowest in the table. These characteristics make it more appropriate and suitable to select among available options for the research.

Table 2. GARCH modeling before the structural break

Model	Coeff	St Error	z-value	p-value	Sig	AIC	BIS
Arch (1)	0.99	0.14	7.01	0.00	***	-1234.22	-1224.29
Garch (1)	0.65	0.02	28.02	0.00	***	-1261.03	-1247.80
TGARCH	0.46	0.11	4.06	0.00	***	-1268.50	-1251.95
EGARCH	0.93	0.01	69.37	0.00	***	-1276.13	-1259.59
SGARCH	0.00	0.00	4.50	0.00	***	-1272.75	-1256.21

*** indicates significant at 10% level, ** indicates significant at 5% level and * indicates that at 1% level

Table 3 demonstrates the various GARCH models applied to the data after the structural (volatility) break. The range of the data is from December 2016 to November 2020. The table represents the various GARCH models, including ARCH, GARCH, TGARCH, EGARCH, and SGARCH. The outcomes exhibited that none of the models is significant at a 1% significance level. The SGARCH is the only model, which is significant at a 5% significance level, while arch and EGARCH are significant at 10%, but the rest of the models are insignificant. This is due to a change in Pakistan's regime, and the new policymakers devalue the currency very rapidly. Also, the increase in inflation and government debts are the other significant reasons for the change in the country's volatility. In this situation, the only significant SGARCH model is selected based on the lowest AIC and BIC criterion to examine the exchange rate volatility after the structural break in data. GARCH model presents healthier results for persistent and smooth change volatility (Chen et al., 2013).

Table 3. GARCH modeling after the structural streak

Model	Coeff	St Error	z-value	p-value	Sig	AIC	BIS
Arch (1)	1.5285	0.8191	1.8700	0.0620	*	-126.53	-121.11
Garch (1)	-0.0141	0.0637	-0.2200	0.8250		-124.71	-117.48
TGARCH	5.3343	3.2960	1.6200	0.1060		-133.37	-124.34
EGARCH	0.4874	0.2892	1.6900	0.0920	*	-130.95	-121.92

SGARCH	0.0878	0.0362	2.4200	0.0150	**	-138.55	-131.32
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*** indicates significance at the 10% level, ** indicates significance at 5% level, and * indicates that at 1% level

In Table 4, the whole range of monthly data is used by eliminating the effect of a structural break. The table denotes that the various GARCH models, including ARCH, GARCH, TGARCH, EGARCH, and SGARCH are applied to estimate the exchange rate volatility. The results display that all of the models are significant at a 1% significance level. The TGARCH model is a more accurate model to predict stock exchange volatility (Munir & Ching, 2019). The TGARCH model elaborates on the mortgage risk price of houses and captures the house price volatility accurately (Lee et al., 2015; Hongwiengjan & Thongtha, 2020). As per the AIC and BIS criteria, TGARCH is the most suitable choice with the lowest AIC and BIS with -1438.02 and -1413.37 values, respectively. Hence, TGARCH is considered a more suitable model to estimate the volatility of the exchange rate within a given time, along with a coefficient value of 0.34. Empirical results showed that threshold GARCH is one of the best models to estimate Bitcoin volatility (Gyamerah, 2019).

Table 4. GARCH modeling without structural break

Model	Coef	St Error	z-value	p-value	Sig	AIC	BIS
Arch (1)	0.61	0.11	5.40	0.00	***	-1361.19	-1350.63
Garch (1)	0.73	0.02	29.45	0.00	***	-1418.01	-1403.92
TGARCH	0.34	0.08	4.01	0.00	***	-1438.02	-1413.37
EGARCH	-0.27	0.09	-2.98	0.00	***	-1366.42	-1348.81
SGARCH	0.00	0.00	5.75	0.00	***	-1364.76	-1343.63

*** indicates significant at 10% level, ** indicates significant at 5% level and * indicates that at 1% level

Figure 3 displays the time series (TS) line of Pakistan's exchange rate in two different graphs. Time is taken on the x-axis while the exchange rate (USD/PKR) is on the y-axis. The figure presents that in the raw form, the exchange rate data is non-stationary, as shown in the upper part of the figure. However, after the transformation, the data becomes stationary, as revealed in the lower part of Figure 3. The figure shows the volatility clustering effect as low volatility is followed by low and high volatility is followed by high volatility for a prolonged period. There is a volatility break in the data shown by a line on the x-axis in December 2016. After the structural break, the data is more volatile than before. It is due to the factors affecting the exchange rate, such as an increase in the inflation rate & government debts and devaluation of the exchange rate. Bala and Asemota (2013) observed the volatility of the exchange rates by using GARCH models. They concluded that the results of the analysis were improved when volatility breaks are considered in the model.

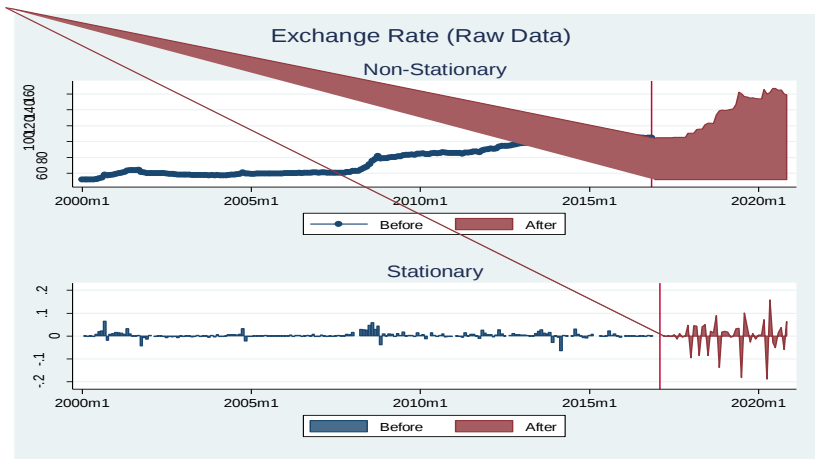


Figure 3. Exchange data (raw data) stationary

Figure 4 shows the four different GARCH models, including GARCH, TGARCH, EGARCH, and SGARCH applied to the exchange rate's monthly data. Figure 4 showing a structural break in December 2016 with a line on the x-axis. The results depict that all of the said GARCH models are significant, and TGARCH presents accurate results. Hence, TGARCH is selected based on the lowest value of AIC and BIS criteria. Empirical results showed that threshold GARCH is one of the best models to estimate Bitcoin volatility (Gyamerah, 2019). The exchange rate fluctuations are limited before the break. However, after the change in government regime, there is enormous volatility shown in the exchange rate—also, TGARCH representing the volatility better than the rest of the GARCH models.

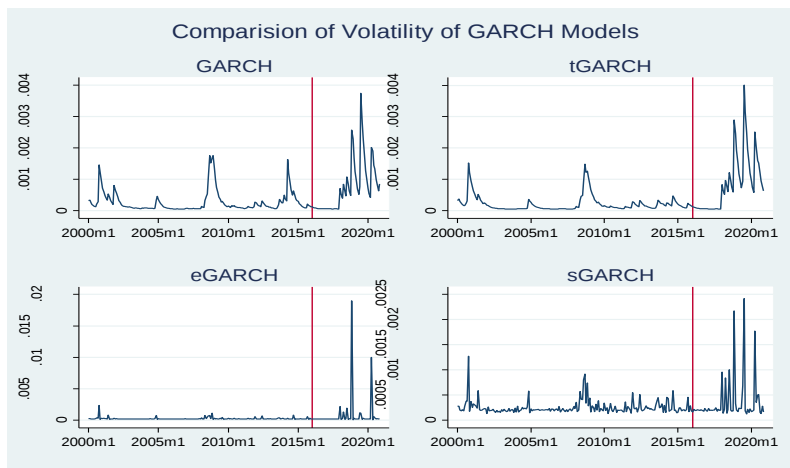


Figure 4. Volatility comparisons GARCH models

EGARCH, and SGARCH. Time and variance are taken on the x-axis and the y-axis, respectively. The models are applied to the whole monthly data of exchange rate for 21 years. Figure 5 shows that TGARCH is expressing more volatility than its family members. That is why TGARCH is chosen on the lowest AIC and BIS criteria. The threshold GARCH model is considered a more accurate model to predict stock exchange volatility (Munir, & Ching, 2019). The TGARCH model elaborates on the mortgage risk price of houses and captures the house price volatility accurately (Lee et al., 2015; Hongwiengjan & Thongtha, 2020).

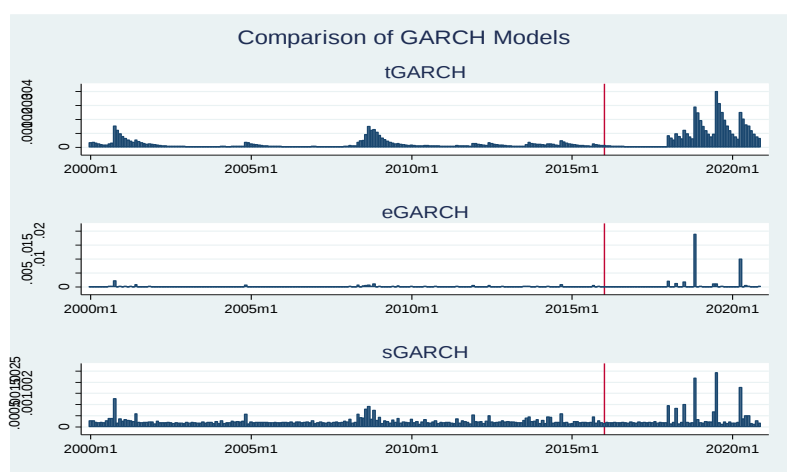


Figure 5. Comparison of GARCH model

DISCUSSION AND CONCLUSION

Raw monthly data of exchange rate is divided into two parts based on the structural break. The first part is starting from an initial point, January 2000 to December 2016 and similarly, the second part is starting from December 2016 to the end of the dataset. Bala and Asemota (2013) concluded that the analysis results were improved when volatility breaks are considered in the model by using the GARCH models. Before the structural break, the exchange rate volatility is lower than that after the structural break due to clustering volatility. Low volatility is followed by low volatility, and high volatility is followed by high volatility for a prolonged period. This phenomenon is due to a change in government regime, an increase in government debts, and Pakistan's inflation rate.

Before the structural break, the EGARCH model is found appropriate to measure the exchange rate volatility. Furthermore, EGARCH has the highest z-value, which is 69.37, and its standard error is the lowest in the table. These characteristics make it more appropriate and suitable to select among

available options. On the contrary, after the break, the only significant SGARCH model is selected based on the lowest AIC and BIC criterion to examine the volatility of exchange rate after the structural break in data. GARCH model presents healthier results for persistent and smooth change volatility (Chen et al., 2013).

Low volatility is followed by low and high volatility is followed by high volatility for a prolonged period. There is a volatility break in the data shown by a line on the x-axis in December 2016 in figure 05. After the structural break, the data is more volatile than before. It is due to the factors affecting the exchange rate, such as an increase in the inflation rate & government debts and devaluation of the exchange rate. The results depict that all of the said GARCH models are significant, and TGARCH presents accurate results. Hence, TGARCH is selected based on the lowest value of AIC and BIS criteria. Empirical results showed that threshold GARCH is one of the best models to estimate Bitcoin volatility (Gyamerah, 2019). It is evident that the exchange rate fluctuations are limited before the break. However, after the change in government regime, there is enormous volatility shown in the exchange rate—also, TGARCH representing the volatility better than the rest of the GARCH models. TGARCH is expressing more volatility than its family members. That is why TGARCH is chosen on the lowest AIC and BIS criteria. The threshold GARCH model is considered a more accurate model to predict stock exchange volatility (Munir, & Ching, 2019).

Finally, this research focuses on the volatility of the monthly exchange rate dataset for 21 years, and the results are consistent with previous studies. This study tries to extend the existing literature, and the paper aims to estimate the volatility of the exchange rate using various GARCH combinations. It is found that the threshold GARCH model is more suitable to estimate the volatility of the time series economic factors such as exchange rate. The TGARCH model is considered a more accurate technique to predict stock exchange volatility (Munir & Ching, 2019). The TGARCH model elaborates on the mortgage risk price of houses and captures the house price volatility accurately (Lee et al., 2015; Hongwiengjan & Thongtha, 2020). On the contrary, if data is broken down based on structural (volatility) break, then the low and high volatility period is estimated with EGARCH and SGARCH, respectively. This study adds to existing literature that structural break significantly impacts estimating the exchange rate volatility. As recommended by the research, it is evident that if data is broken down based on structural break, then the low and high volatility period is estimated with EGARCH and SGARCH, respectively. Nevertheless, on the other hand, if the whole data without a break is considered, the threshold GARCH model is more appropriate to estimate the volatility of the time series economic factors such as exchange rate.

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